6

Interaction of Radiations with Matter

6.1 Introduction

Nuclear radiations may be classified into the following *three* types.

1. ***Chargeless radiations*** like X-rays, γ-rays.
2. ***Neutral particles*** such as neutrons and neutrinos.
3. ***Charged particles*** such as electron, proton, α-particles and heavier ions.

Charged particles may further be classified as *heavy charged particles*, e.g. protons, α-particles, heavy ions and *light charged particles* such as electrons, positrons and charged muons. Electrons and positrons are characterised by very small masses.

All radiations, irrespective of the nature of their charge, lose energy while passing through matter. The amount of energy lost by radiation is *radiation-specific* and hence is used in designing *radiation detectors* and is also useful for *radiation therapy* and *dosimetry*. It is thus very important to study the processes of radiation-loss in passing through different materials and the present chapter is devoted to the same.

When a beam of any radiation penetrates matter, some of it may be *absorbed completely*, some may get *scattered* and some again may pass straight *without any interaction*. The absorption and scattering can be explained by interaction between the incident radiation and the electrons, atoms and molecules of the target material.

There is again *two types of processes* of energy-loss: in one, the energy-loss is *gradual*, almost *continuous* and through *many interactions*; in the other, the energy-loss is *catastrophic*, the incident radiation moves *without any interaction* in the material until it loses all its energy, in a *single collision*—a *one-shot* process.

6.2 Interaction of γ-rays with matter

If γ-radiation passes through an absorber, its intensity I changes. The change in intensity ΔI is proportional to the intensity I of γ-rays and the absorber thickness Δx.

$$\therefore \quad \Delta I \propto -I\,\Delta x \;\Rightarrow\; \Delta I = -\mu I \Delta x, \tag{6.2.1}$$

the negative sign indicates that as x increases, I decreases. The constant of proportionality $\boldsymbol{\mu}$ is called the ***absorption coefficient*** or ***attenuation coefficient***. For a given absorber, μ depends on the energy of γ-radiation.

Re-writing (6.2.1) as: $\Delta I/I = -\mu\,\Delta x$, and integrating, we get

$$\ln I = -\mu x + c \tag{6.2.2}$$

At $x = 0$, $I = I_0$, the initial intensity of γ-rays. So, from (6.2.2), $c = \ln I_0$.

$$\therefore \quad \ln I = -\mu x + \ln I_0 \;\Rightarrow\; \ln(I/I_0) = -\mu x \tag{6.2.3}$$

$$\therefore \quad \boxed{I = I_0 e^{-\mu x}} \tag{6.2.4}$$

Eq. (6.2.4) shows that the intensity of γ-rays *decreases exponentially* with x and is never zero. The interaction of γ-rays with matter is quite different from that of charged particles. γ-rays have greater penetrating power and different absorption laws.

● *The absorption coefficient* μ is also called ***linear absorption coefficient***. The quantity $\mu_m = \mu/\rho$, where ρ is the density of the absorber, is *defined* as ***mass absorption coefficient***. It can be shown that

$$\mu = \frac{\sigma N_A \rho}{A}$$

where A = the mass number and N_A = Avogadro number.

● The quantity μx in (6.2.4) is obviously dimensionless. So, if x is expressed in metre, then μ will have the dimensions m^{-1}.

● If $x = 1/\mu$, then from (6.2.4), $I = I_0/e$. Hence, the ***linear absorption coefficient*** may also be *defined* as the *reciprocal of that thickness of material that reduces the intensity of incident γ-rays to $1/e$ of its initial value.*

6.2.1 Two more terms

We now define *two* more terms associated with *γ-ray absorption*. These are :

1. Radiation length—The ***radiation length***, also called the ***absorption length***, is *defined as the reciprocal of the linear attenuation coefficient* μ. It is thus *the thickness of the material that reduces the intensity of the incident radiation to* $1/e$ *of its initial value.*

2. Half-thickness—The ***half-thickness*** of an absorber is *defined as that thickness that reduces the intensity of γ-radiation to one half of its initial value.*

From (6.2.4), therefore, $I = I_0/2$ for $x = x_{1/2}$, the *half-thickness.*

$$\therefore \quad \mu x_{1/2} = \ln 2$$

where $x_{1/2}$ is the *half-thickness*, i.e., the thickness reducing the intensity by half of the original value.

$$\therefore \quad \boxed{\mu = \frac{\ln 2}{x_{1/2}} = \frac{0.6931}{x_{1/2}}}$$

6.2.2 Experimental determination of μ

The linear attenuation coefficient μ is determined experimentally by using (6.2.3) :

$$-\mu x = \ln(I/I_0)$$

This shows that if $\ln(I/I_0)$ be plotted against the thickness x, a *straight line* would be obtained whose *slope* is the *attenuation coefficient*, $\boldsymbol{\mu}$.

The ***experimental arrangement*** is illustrated by block diagram in Fig. 6.1. The radioactive source S that emits γ-rays, is enclosed in a thick Pb-block with a fine opening on one side. A very fine beam of γ-rays comes out of the opening, passes through a Pb-collimator A and finally through the absorber B, whose μ-value is to be measured. C is another Pb-collimator that stops the photons scattered from absorber B. Finally, the γ-beam, attenuated and transmitted, reaches the the heavy-shielded γ-detector D.

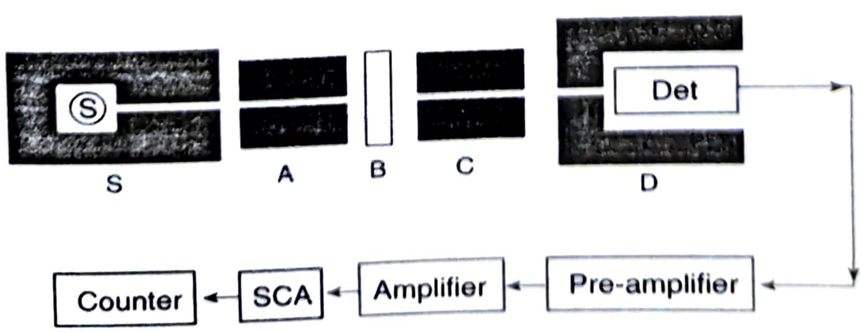

Fig. 6.1 Experimental arrangement for determining μ (in block diagram)

First, the experiment is conducted *without the absorber* in position B and counts are recorded in γ-ray spectrometer. This gives I_0. Next, absorbers of different thicknesses are placed at B and the corresponding counts I are recorded. A plot of $\ln(I/I_0)$ vs thickness x of the absorber, as shown in Fig. 6.2, gives a *straight line*. The *slope* of the straight line is the μ-value.

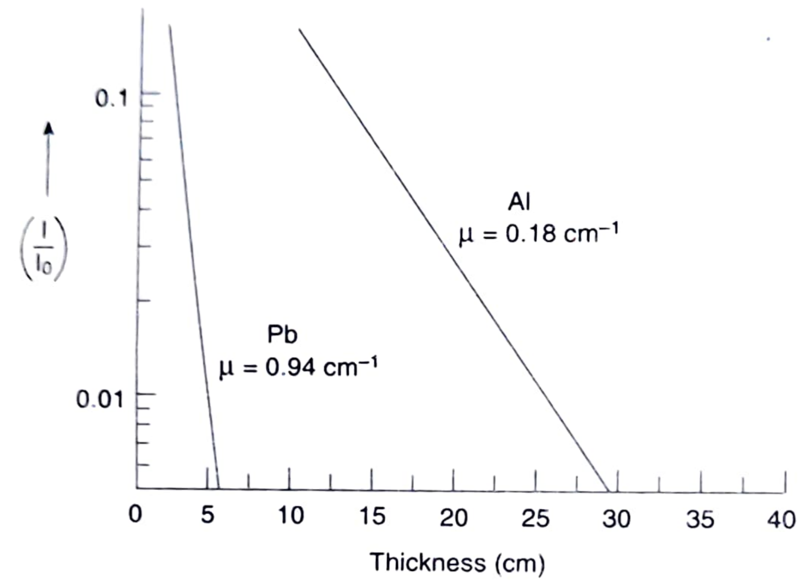

Fig. 6.2 ln (I/I_0) vs absorber thickness. Plot is on semilog scale and 'ln' on y-axis omitted

- For Al, $\mu = 0.18\,\text{cm}^{-1}$, for Pb, $\mu = 0.94\,\text{cm}^{-1}$. The μ-value is larger for heavier elements and conversely. It also depends on the energy of γ-rays—larger for lower energies and conversely.

6.2.3 Dominant modes of energy-loss by photons

Although, in principle, there may be many different ways in which γ-photons may interact with atoms, atomic electrons and atomic nucleus, the following *three* processes are the dominant modes of energy loss of γ-radiation.

1. The ***photoelectric effect***,
2. The ***Compton effect***, and
3. ***Pair production***.

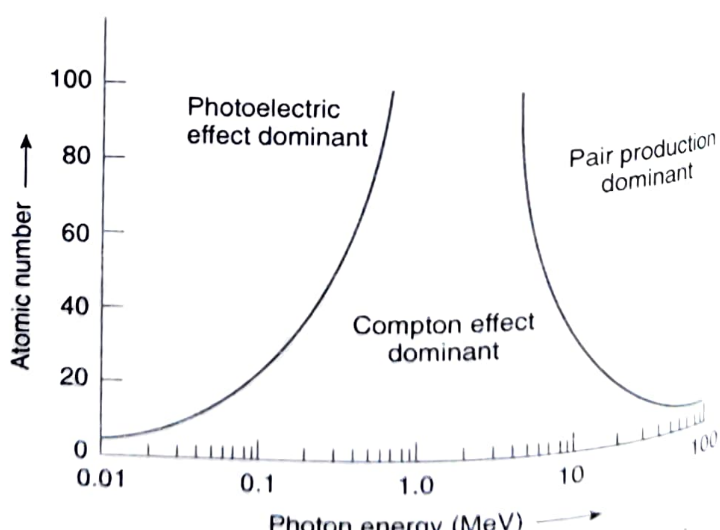

Fig. 6.3 Regions of dominance of different energy-loss processes as a function of photon energy

Each of the above interactions, peaks and dominates in a different range. This is illustrated in Fig. 6.3.

The probability of each process is expressed by the corresponding absorption coefficient. The *total*

absorption coefficient μ_T *is the sum of the absorption coefficients of each of the three processes*, i.e.,

$$\mu_T = \mu_{pe} + \mu_\sigma + \mu_{pp}$$

where the indices pe, σ and pp refer to the photoelectric effect, the Compton effect and the pair production respectively.

6.3 Photoelectric effect

In ***photoelectric effect***, the incident photon of energy $h\nu$ interacts with the *bound electron* of the atom, ejects it out and loses all its energy to disappear in *one shot*. A certain *minimum energy*, the binding energy B_e, also called the *work function* ϕ, is

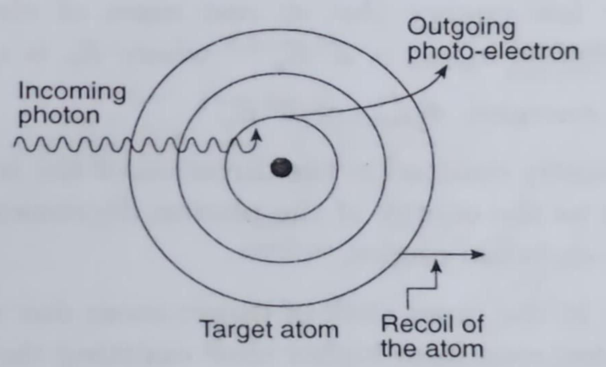

Fig. 6.4 Schematic representation of photoelectric effect

required to dislodge the electron from the binding of the atom. The ejected electron, called ***photoelectron*** (Fig. 6.4), thus moves with a kinetic energy E given by

$$E = h\nu - B \tag{6.3.1}$$

$$\text{or,}\quad \tfrac{1}{2}mv^2 = h\nu - \phi \tag{6.3.2}$$

When the entire energy of the photon is used up in ejecting the electron and nothing is left out, $h\nu - \phi = 0 \Rightarrow \phi = h\nu_0$, where ν_0 is the corresponding incident frequency—the *threshold frequency*. So, from (6.3.2)

$$\tfrac{1}{2}mv^2 = h(\nu - \nu_0) \tag{6.3.3}$$

Equation (6.3.2) and (6.3.3) are two *alternative* expressions for Einstein's ***photo-electric equation*** which has been well verified experimentally.

The incident photon brings in a linear momentum $h\nu/c$ and the target atom recoils to conserve the linear momentum. But, due to the larger mass of the atom, this recoil energy is too small and is neglected. Although the recoil energy is disregarded, the momentum and energy conservations must hold for the photoelectric effect to occur. The probability for recoil of the atom depends on how tightly the photoelectron is bound to the atom. As the K-shell electron is most tightly bound, the probability of photoelectric effect is greatest with K-electrons.

- As the target atom loses the photoelectron, it gets ionized. So, the photoelectric effect is also termed ***photo-ionization***.

If the incident energy of the photon becomes very high, the binding energy of the electron may be insignificant. So, to a high-energy photon, each electron of the atom will appear as *free, unbound*. As a result, the probability of photoelectric effect, which requires a bound electron for momentum conservation, will decrease with increasing photon energy. And the photoelectric effect would give rise to *another process* the ***Compton effect*** or ***Compton scattering***, discussed in Art. 6.4.

Quantum mechanical treatment of photoelectric effect is much involved and beyond our scope. But the *following conclusions* may be drawn by analysing the experimental and the theoretical data.

1. For photons of very low energy ($h\nu \ll$ rest mass of electron but $> B$), the *photoelectric cross-section*, $\sigma_{\text{photo}} \propto Z^5 E_\nu^{-3.5}$ where E_ν is the photon energy.
2. At very high photon energies, $\sigma_{\text{photo}} \propto Z^5 E_\nu^{-1}$.
3. Photoelectrons are mostly emitted in the direction of the incident photon at high photon energies. But as the energy of the photon decreases, the photon emission peak shifts to higher emission angles.
4. The electron vacancy in the inner shell of target atom due to photoelectric effect, may be filled up by electrons from higher shell emitting the *characteristic X-rays* or *Auger electrons*.

   The ion, before capturing an electron from the surroundings, may produce ionization or excitation of nearby atoms of the material.
5. The ***mass attenuation coefficient*** for photoelectric absorption *decreases* with ***increasing photon energy*** i.e., high energy photons are more penetrating than low energy ones. For a ***fixed value of energy***, the ***attenuation coefficient*** *increases* with the Z-value of the material.

- For more details on '*Photoelectricity*', read 'Modern Physics' by A.B. Gupta.

6.4 Compton scattering

If the energy of the incident photon (or γ-ray) be much greater than the binding energy of electron in the atom, then the photon gets scattered by the atomic electron considered *free* and unbound. The incident photon transfers a part of its energy to the electron and itself gets scattered with reduced energy. The scattered photon with less energy than original photon, is thus different from the original one and is removed from the beam in a single collision with the electron. ***Compton scattering*** is thus also a *one-shot* process.

The probability of Compton scattering is more with most weakly bound electron, that is, *valence electron*. Both the energy and the momentum are conserved in Compton

scattering between the incident photon,the scattered electron and the scattered photon. We shall utilize this in computing the *energy-loss of photon* by way of energy transfer to electron. Schematically, the scatting process is shown in Fig. 6.5.

Calculation of energy transfer—Let the energy E of the photon *before* collision be $h\nu$. So, its momentum p *before* collision is $h\nu/c$, c being the velocity of light in vacuo. Let the energy E' of the photon *after* collision be $h\nu'$. So, its momentum p' *after* collision is $h\nu'/c$.

For simplicity, we take the electron *at rest*, prior to collision. So, its energy $E_1 = m_0c^2$ corresponding to rest mass m_0 and its momentum p_1 is zero. Let v be be its velocity *after* collision so that its momentum p_1' is mv and its energy $E_1' = mc^2$.

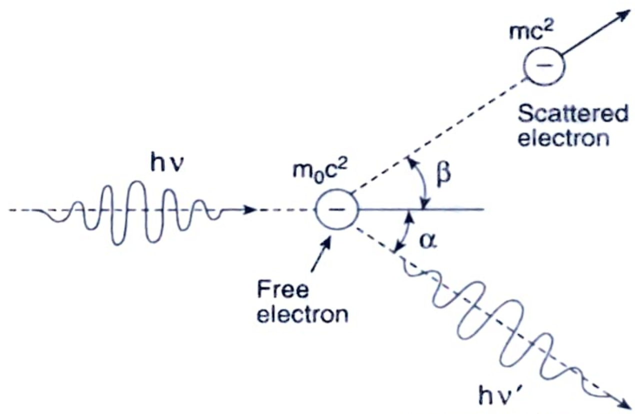

Fig. 6.5 Schematic repressentation of Compton scattering

Let α be the *angle of scattering* of photon and β the *angle of deviation* of electron. Then, the laws of *conservation of momentum and energy* dictate:

$$\left.\begin{aligned} h\nu/c &= (h\nu'/c)\cos\alpha + mv\cos\beta \\ 0 &= (h\nu'/c)\sin\alpha - mv\sin\beta \end{aligned}\right\} \quad \text{momentum conservation} \tag{6.4.1}$$

$$\text{and} \quad h\nu + m_0c^2 = h\nu' + mc^2 \quad \text{Energy conservation} \tag{6.4.2}$$

From (6.4.1)

$$\left.\begin{aligned} mvc\cos\beta &= h(\nu - hu'\cos\alpha) \\ mvc\sin\beta &= h\nu'\sin\alpha \end{aligned}\right\} \tag{6.4.3}$$

Squaring the equations in (6.4.3) and adding, we get

$$m^2v^2c^2 = h^2(\nu^2 + \nu'^2 - 2\nu\nu'\cos\alpha) \tag{6.4.4}$$

From (6.4.2) again, $\quad mc^2 = h(\nu - \nu') + m_0c^2$

$$\Rightarrow \quad m^2c^4 = h^2(\nu^2 + \nu'^2 - 2\nu\nu') + 2m_0c^2h(\nu - \nu') + m_0^2c^4 \tag{6.4.5}$$

Subtracting (6.4.4) from (6.4.5), we obtain

$$m^2c^4(1 - v^2/c^2) = -2h^2\nu\nu'(1 - \cos\alpha) + 2m_0c^2h(\nu - \nu') + m_0^2c^4$$

$$(\because \; m = m_0/\sqrt{1 - v^2/c^2})$$

$$\therefore \quad 2h^2\nu\nu'(1-\cos\alpha) = 2m_0c^2h(\nu-\nu')$$

$$\text{or,} \quad (1-\cos\alpha) = \frac{m_0c^2}{h}\cdot\frac{\nu-\nu'}{\nu\nu'} = \frac{m_0c^2}{h}\left(\frac{1}{\nu'}-\frac{1}{\nu}\right) \tag{6.4.6}$$

$$\text{But } d\lambda = \lambda' - \lambda = c\left(\frac{1}{\nu}-\frac{1}{\nu}\right)$$

$\therefore$ From (6.4.6) $\quad \boldsymbol{d\lambda = \lambda - \lambda =}$ ***Compton-shift***

$$\text{or,} \quad \boxed{\boldsymbol{d\lambda = \frac{h}{m_0c}(1-\cos\alpha)}} \tag{6.4.7}$$

Energy of scattered photon—From (6.4.6), we have

$$\frac{1}{\nu'}-\frac{1}{\nu} = \frac{h}{m_0c^2}(1-\cos\alpha)$$

$$\therefore \quad \nu' = \frac{1}{\dfrac{1}{\nu}+\dfrac{h}{m_0c^2}(1-\cos\alpha)}$$

$$\therefore \quad \boldsymbol{h\nu' = \frac{h\nu}{1+\dfrac{h\nu}{m_0c^2}(1-\cos\alpha)}} \tag{6.4.8}$$

Energy of recoil electron—Since the *gain in electron-energy is equal to photon energy*, the *kinetic energy* of recoil electron is

$$E_k = h\nu - h\nu' = h\nu - \frac{h\nu}{1+\dfrac{h\nu}{m_0c^2}(1-\cos\alpha)}$$

$$= h\nu\frac{\boldsymbol{(h\nu/m_0c^2)(1-\cos\alpha)}}{\boldsymbol{1+(h\nu/m_0c^2)(1-\cos\alpha)}}, \quad \text{on simplication.} \tag{6.4.9}$$

Maximum energy transferred to electron—This is, from (6.4.9), is given by

$$\boldsymbol{E_{\max} = \frac{h\nu}{1+\dfrac{m_0c^2}{h\nu}}} \tag{6.4.10}$$

Observations—The following observations from the above set of equations (6.4.7) through (6.4.10) are noteworthy.

1. *Compton-shift in wavelength* in any direction is *independent of the incident energy of photon.*
2. *Compton-shift in energy*, however, is *strongly dependent on the incident energy of photon.*
3. *High energy photon suffers a very large change in energy*, while low *energy photon suffers a moderate change.*

4. The value of h/m_0c involves only the fundamental constants h, c and m_0, and is thus a *universal constant*, having the dimensions of *length*. It is called ***Compton wavelength*** λ_c for electron. Its value is 0.024 A.

The energy distribution of scattered electrons for two different energies of incident photons, as illustrated in Fig. 6.6, shows that the *number of scattered electrons remains almost the same for all energies below the energy of incident γ-rays*; thereafter, it *slightly increases towards the higher energy end* and finally *drops abruptly to zero just below* the *energy* of the *incident photon*. The *rise of the energy distribution curve at higher energy end* is termed ***Compton edge***.

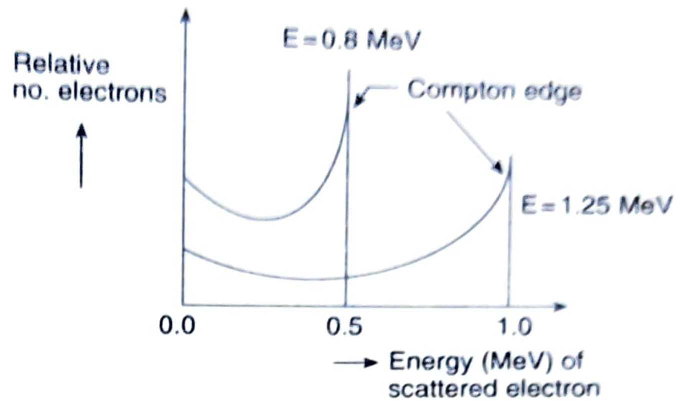

Fig. 6.6 Energy distribution of scattered electrons

- In photoelectric effect, each γ-photon produces one photoelectron and one recoiling ion. In Compton effect, each incident γ-photon produces one electron, one scattered γ-photon of lower energy and an ion of the target atom. So, the number of radiations after the interaction increases in both the effects. This increase in the number of radiations plays a vital role in the *design of radiation shielding.*

Using Dirac's relativistic electron equation, Klein and Nishina gave the following expression for Compton scattering cross-section per electron σ_{comp}.

$$\sigma^e_{\text{comp}} = \left(\frac{\pi^2 e^2}{4\pi\varepsilon_0 m_e c^2}\right)^2 \left[\left(1 - \frac{2(\gamma+1)}{\gamma^2}\right)\ln(2\gamma+1) + \frac{1}{2} + \frac{4}{\gamma} - \frac{1}{2(2\gamma+1)^2}\right]$$

where $\gamma = h\nu/m_0c^2$.

The total cross-section per atom, $\sigma^{\text{atom}}_{\text{comp}} = Z\sigma^e_{\text{comp}}$, where Z = number of electrons per atom. Note that Compton scattering cross-section for a material is proportional directly to the atomic number Z of the material and inversely to the energy of the incident γ-ray.

For more details on '*Compton effect*', consult 'Modern Physics' by A.B. Gupta.

6.5 Pair production

Apart from photoelectricity and Compton effect, there is a *third* possibility in which a γ-photon materialises into an electron and proton.

Gamma rays of energy $E_\gamma > 1.02$ MeV may, by interaction with atom, lose all its energy and produce a pair of electron and positron. This phenomenon is called ***pair production***. Symbolically, we may write

$$\text{photon} \rightarrow \text{electron} + \text{positron}$$

$$\gamma \rightarrow e^- + e^+$$

All the conservation laws are fulfilled during the process. Sum of the charge of electron ($-e$) and of position ($+e$) is zero and the charge of photon is also zero. The linear momentum is also conserved with the help of the *nucleus of target atom* that carries away enough photon momentum for conservation. The total rest mass energy of electron and positron is $m_0c^2 + m_0c^2 = 2m_0c^2 = 2 \times 0.51\,\text{MeV} = 1.02\,\text{MeV}$. This is the minimum energy the incident photon must possess for the process to occur. If γ-ray possesses energy $> 1.02\,\text{MeV}$, the excess energy is shared by the pair as kinetic energy. The pair production process is illustrated schematically, in Fig. 6.7(a).

The exact theory of pair production can be derived only from the field theory. But it may as well be understood from Dirac's theory of the sea of negative energy electrons. Dirac showed that the energy spectrum of a free electron consists of two branches. One starts at $+m_0c^2$ and extends to $+\infty$ and the other starts at $-m_0c^2$ and extends to $-\infty$, as momentum $|p| \to \infty$. A forbidden gap of $2m_0c^2$ separates the two branches (Fig. 6.7b). In this theory, vacuum is considered to be filled with

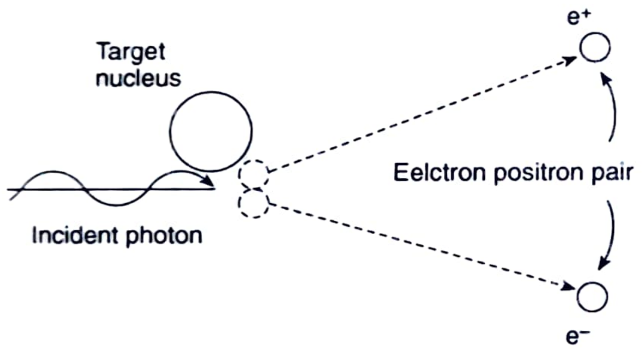

Fig. 6.7(a) Schematic representation of pair production

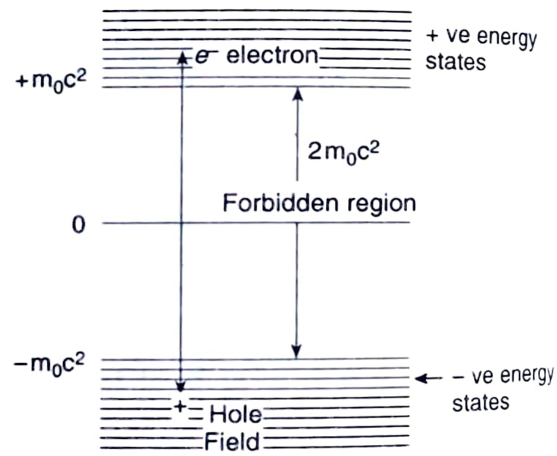

Fig. 6.7(b) Dirac diagram for electron and positron production

electrons of all negative energy and cannot be detected. On interaction of a negative energy electron with an electromagnetic field, it goes to a positive energy state and becomes an ordinary *electron*. The empty space remaining in the midst of negative energy, manifests itself as a particle of the *same mass* but with *positive charge* and is called a ***positron*** and often a *hole*.

- Pair production is possible only ***if***

1. The energy of the incident photon is greater than or equal to $2m_0c^2$, the sum of the rest mass energies of electron and positron. The energy in excess of $2m_0c^2$ is shared as kinetic energy by the pair.

2. There is the presence of a field like that of a target nucleus which is required for conservation of momentum. Pair production cannot occur in free space (vacuum).

The cross-section for pair production, σ_{pair} for the nucleus of atomic number Z is

given by

$$\sigma_{\text{pair}} = \frac{Z^2}{137}\left(\frac{e^2}{4\pi\varepsilon_0 m_0 c^2}\right)\left[\frac{28}{9}\ln\frac{2h\nu}{m_0c^2} - \frac{218}{27}\right]$$

From above, the probability of pair production (and hence the mass absorption coefficient) is proportional to the square of the atomic number Z and to $\ln E_\gamma$ for γ-rays of higher energies.

• The end products of pair production are two charged particles: e^- and e^+ and a recoiling nucleus. Again, the number of radiations after the process is more than those before the process.

6.5.1 Pair annihilation

A *reverse process* occurs when a positron comes close to an electron under the influence of their opposite charges. They annihilate each other to vanish simultaneously giving rise to two γ-photons of energy > 0.51 MeV each, moving in opposite directions:

$$e^- + e^+ \rightarrow \gamma + \gamma.$$

This process is known as ***pair annihilation***. The two γ-rays move in opposite directions to conserve the linear momentum. For this annihilation process, the presence of nucleus (for momentum conservation) is not required and hence may occur in vacuum as well.

6.6 Energy-loss by chargeless nuclear particles

Chargeless nuclear particles are of two kinds; (i) *heavy*, e.g., neutrons and (ii) *very light*, e.g., neutrinos. Their energy-loss processes are discussed with *neutrons* and *neutrinos* as representatives.

Energy loss by neutrons—The energy-loss by neutrons depend both on the energy of the neutron and on the properties of the absorbing material. Neutron interactions are broadly of *two* types.

1. Interaction with the atom (or nucleus) of the absorber and the loss of a part of its energy. It remains however in the system.

2. Interaction with the nucleus of the target atom to initiate a nuclear reaction and its disappearance from the system.

The first type is called ***scattering*** where the process of energy-loss is similar to that of heavy charged particles. The second type is like a '***one-shot***' *process* of losing energy.

Scattering, again, can be both *elastic* and *inelastic*. In *elastic scattering*, the total kinetic energy of neutron and the nucleus remains unaltered and a fraction of neutron's K.E. gets transferred to the nucleus. The *average energy-loss* by neutron is $2AE/(A+1)^2$, where E is the K.E. of the neutron and A the atomic weight of target nucleus. The relation shows that to reduce the neutron-speed (i.e., *moderation*) with

minimum number of elastic collisions, the A-values of target nucleus should be small. With hydrogen, $A = 1$, the average energy-loss is highest, $E/2$. So, with H-nucleus, 2 MeV neutron will have 1 MeV left after one collision, 0.5 MeV after the second collision and so on. In general, if after n collisions, the K.E. of neutron changes from initial value E_0 to E_n,

$$E_n = E_0 \left[\frac{A^2+1}{(A+1)^2} \right]^n$$

$$\Rightarrow \quad n = \frac{\ln(E_n/E_0)}{\ln \dfrac{A^2+1}{(A+1)^2}}, \quad \text{on an average.}$$

The *inelastic scattering* is similar to elastic scattering except that the nucleus gets an internal rearrangement and shots into an excited state to release eventually radiation. In this scattering, the total K.E. of the outgoing neutron and of the nucleus is less than the K.E. of the incoming neutron; a part of the K.E. of neutron is used to shot the nucleus into the excited state. If the excited state of the nucleus is too high (energetically) to reach with the available energy from incoming neutron, inelastic scattering is *not possible*. H-nucleus has no excited states and so only elastic scattering can occur. In general, scattering reduces the energy of neutrons and provides the basis of some neutron detectors.

Scattering apart, a neutron may be absorbed by the target nucleus to initiate $(n,\gamma), (n,p), (n,d), (n,\alpha), \cdots$, i.e., various types of *nuclear reactions*, depending on the energy of neutron and the target nucleus. An important reaction in this context is $^{113}\text{Cd}\,(n,\gamma)\ ^{114}\text{Cd}$ with a very high cross-section for *thermal neutrons* (0.025 MeV). Cd-rods are therefore used in reactors to remove/absorb thermal neutrons to control the fission rate.

The *intensity* of a collimated neutron beam *falls off exponentially* with the thickness of the absorber, since the incident neutrons from the original beam are removed by nuclei through scattering and absorption for nuclear reaction. The relation is

$$I = I_0 e^{-\mu_m d_m}$$

where I_0, I are respectively the intensities of neutron beam *before* and *after* traversing the thickness d_m (g/cm^2) of the absorber of mass absorption coefficient μ_m. μ_m can be obtained if the cross-sections for scattering and for absorption are known.

Energy loss by neutrino—Neutrinos have no charge, almost zero mass and they interact with matter through weak interactions with very small cross-sections. Hence, the *energy-loss by neutrinos in their passage through matter is extremely small*.

Consider, the nuclear reaction in which a proton captures an antineutrino to produce a neutron and a positron :

$$p + \bar{\nu}_e \to n + e^+$$

The cross-section of this reaction is $\simeq 10^{-43}$ barn and 1 c.c. of matter contains $\simeq 10^{24}$ protons.

So, the ***interaction probability*** is :

$$10^{24}\,(\text{cm}^{-3}) \times 10^{-43}\,(\text{cm}^2) = 10^{-19}\,\text{cm}^{-1}$$

Thus, on an average, an antineutrino will pass through a distance of 10^{19} cm (> 10 light years!) before being captured. This is why, it took an unusually long period for its detection.

• There are *three* kinds of neutrinos : *electron neutrino* (ν_e), *muon neutrino* (ν_μ) and *tau neutrino* (ν_τ). They have different properties and interact with matter with different probabilities and are detected through their reaction products.

6.7 Energy-loss by light charged particles

Electrons, positrons and muons form the *light charged particles* (LCP). They lose their energy by *three* different processes. These are :

1. By ***ionisation and excitation*** of atoms of the target material they pass through, just like heavy charged particles, but with a difference due to their very small masses, (to be discussed in Art. 6.8).

2. By ***bremstrahlung***, when they come near the nucleus of the absorber-atom and get accelerated or retarded, depending on their charge, to emit radiation in the form of X-rays or low frequency e. m. waves, resulting in energy-loss.

3. LCP's of high kinetic energies also emit ***Cerenkov radiation*** in transparent media to lose energy.

We shall discuss the above processes with electron as representative of LCPs.

Energy-loss of electrons by ionisation and excitation. This mode of energy loss is called the ***collision loss***. As the *mass* of the incident electron and the struck electron of the target atom is *equal*, about half of the K.E. of incident electron may be lost by *a single collision* and they may *scatter in any direction*. Also, it is not possible to distinguish between the incident and the struck electron and so the path of the incident electron in the absorber is *not straight* but *zig zag*. The kinetic energies of the incident and the scattered electrons are generally large as more energy is lost per collision. Further, the scattered electrons (δ-rays) may as well *ionize* other atoms in the material.

Radiative energy-loss—When high energy electrons come close to the nucleus of the target atom, they get accelerated. Accelerated charge emits e. m. radiations and thereby loses energy. This is termed ***radiative loss***.

The probability of radiative loss is proportional to the square of the acceleration. And hence high energy electrons that may reach very close to the target nucleus may *lose substantial amount of energy*.

Energy loss by Cerenkov radiations—Light charged particles, with speed v, greater than that of light in a transparent medium (c/μ), emit Cerenkov radiations and thereby lose energy. The required condition for emission of Cerenkov radiation is $v > (c/\mu)$ or $\beta\mu > 1$, where μ is the index of the medium and $\beta = v/c$.

The threshold energy for such radiations is given by

$$E_{\text{th}} = m_0c^2\left(-1+\sqrt{1+\frac{1}{\mu^2-1}}\right)$$

and they are generated by the interaction of the e. m. field of the incident charged particle with electrons of the transparent material.

● The *energy loss per unit path length* by ***Cerenkov radiation*** is given by

$$\left(\frac{dE}{dx}\right)_{\text{cer}} = \frac{4\pi^2Z^2e^2}{c^2}\int\left(1-\frac{1}{\beta^2\mu^2}\right)\nu\, d\nu,$$

the integration is over all frequencies. The relative contribution from higher frequencies however is dominant.

The *energy-loss* by Cerenkov radiations is *very small*, $\sim$ 1 keV per cm of path length and, *compared to collision-loss*, is in fact *negligible*.

● The importance of Cerenkov radiations however is in their use in detecting relativistic charged particles by Cerenkov detectors. For more details, consult 'Modern Physics' by A.B. Gupta.

Calculation of different types of energy-losses—The calculations for these two types of energy-losses involve advanced quantum mechanics. The final expression for the *linear stopping power* S_e^c of electron due to ***collision*** is given by

$$S_e^c = -\left(\frac{dE}{dx}\right)_c = \left(\frac{e}{4\pi\varepsilon_0}\right)^2\cdot\frac{2\pi e^2NZ}{m_ec^2\beta^2}F$$

where v is the velocity and E the K.E. and m_e the mass of electron, Z the atomic number of stopping medium, I its average ionisation potential, N the number density of atoms in the absorber and $\beta = v/c$, $c =$ velocity of light in vacuo and F is given by

$$F = \left[(1-\beta^2)+\frac{1}{8}\left(1-\sqrt{1-\beta^2}\right)^2-\left(2\sqrt{1-\beta^2}-1+\beta^2\right)^2\ln 2+\ln\frac{m_ev^2E}{2I(1-\beta^2)}\right]$$

As in heavy charged particles (Art 6.8), the *linear stopping power* for electrons is *inversely proportional to the square of velocity or the energy of electron*. With increase of energy, the collision-loss decreases to give way finally to radiative-loss.

For electron of energy $E > m_0c^2$, Bethe and Heitler gave the following expression for ***radiative-loss*** *per unit path length*:

$$S_e^r = -\left(\frac{dE}{dx}\right)_r = \left(\frac{e}{4\pi\varepsilon_0}\right)^2\frac{NZ(Z+1)Ee^2}{137m_e^2c^4}\left[4\ln\frac{2E}{m_ec^2}-\frac{4}{3}\right]$$

The ratio of collision-loss to radiative-loss may roughly be given as

$$\frac{S_e^c}{S_e^r} = \frac{E\,(\text{in MeV})\,Z}{700}$$

The radiation-loss for higher energies increases almost exponentially. At lower electron energies, the collision-loss dominates over the radiative-loss.

6.8 Energy-loss by heavy charged particles

When heavy charged particles pass through an absorbing material, they lose their kinetic energy predominantly by ***ionisation*** and/or ***excitation*** of atoms of the target material. Since an atom has a large number of electrons (e.g., 82 per atom in Pb), the most of the time the collision is with electrons. We shall hence neglect the collisions with atoms as a whole, which are rare. These interactions involve small amount of energy-loss per collision, large number of collisions and almost straight line motion of the incident charged particle. The ionisation and excitation of target atoms occur through Coulomb interaction between the incident charged particle and the electrons. The range of Coulomb force is infinite. So, many electrons may simultaneously interact with the incident charged particle.

The charged particles may also lose their energy by ***bremstrahlung*** in which e. m. radiations (photons) are emitted when charged particles get accelerated or decelerated in the nuclear field of the target atoms.

Another process of energy-loss is by emission of ***Cerenkov radiation*** when a high energy charged particle travels in a medium with a velocity exceeding the velocity of light in that medium.

• The amount of energy-loss in each of the above *three processes* depends on the charge and the energy of the particle and also on the atomic number and the density of the absorbing material.

6.8.1 Maximum energy transfer to electron

The energy-loss process can be treated *classically* with following *assumptions*:

1. Electrons in the target atoms are free and at rest.
2. Energy transfer to the electron is more than its binding energy to the atom.
3. Incident charged particle moves faster than the atomic electrons.

The above assumptions make *the interaction* between the charged particle and the atomic electron *essentially elastic.*

Maximum energy transfer—The maximum energy-loss occurs in ***head on collision*** with electron. Let a particle of rest mass M move with initial velocity v to collide head on with a stationary electron of mass m_e. An electron becomes relativistic at rather low energies, requiring relativistic calculations.

The initial total energy of the particle is $E_p^i = \gamma Mc^2$ where $\gamma = \sqrt{1 - v^2/c^2}$, the Lorentz factor and that of electron, $E_e^i = m_e c^2$.

Initial total energy of (particle + electron) is thus

$$E_i \text{ (total)} = E_p^i + E_e^i = \gamma Mc^2 + m_e c^2$$